INSIDE A SYMPHONY (1974)

By Per Nørgård

In 1971 I was told by the head of the Music Department of the Danish State Radio, Mogens Andersen, that his department intended to commission a television opera from me. This conformed to a long-standing practice of commissioning works from Danish composers, a practice which, in my case, had earlier resulted in the Eugene Ionesco-Flemming Flindt ballet, *Den Unge Mand Skal Giftes* (The Young Man is Going to be Married) from 1964, which was shown on European television (EBU) in 1965.

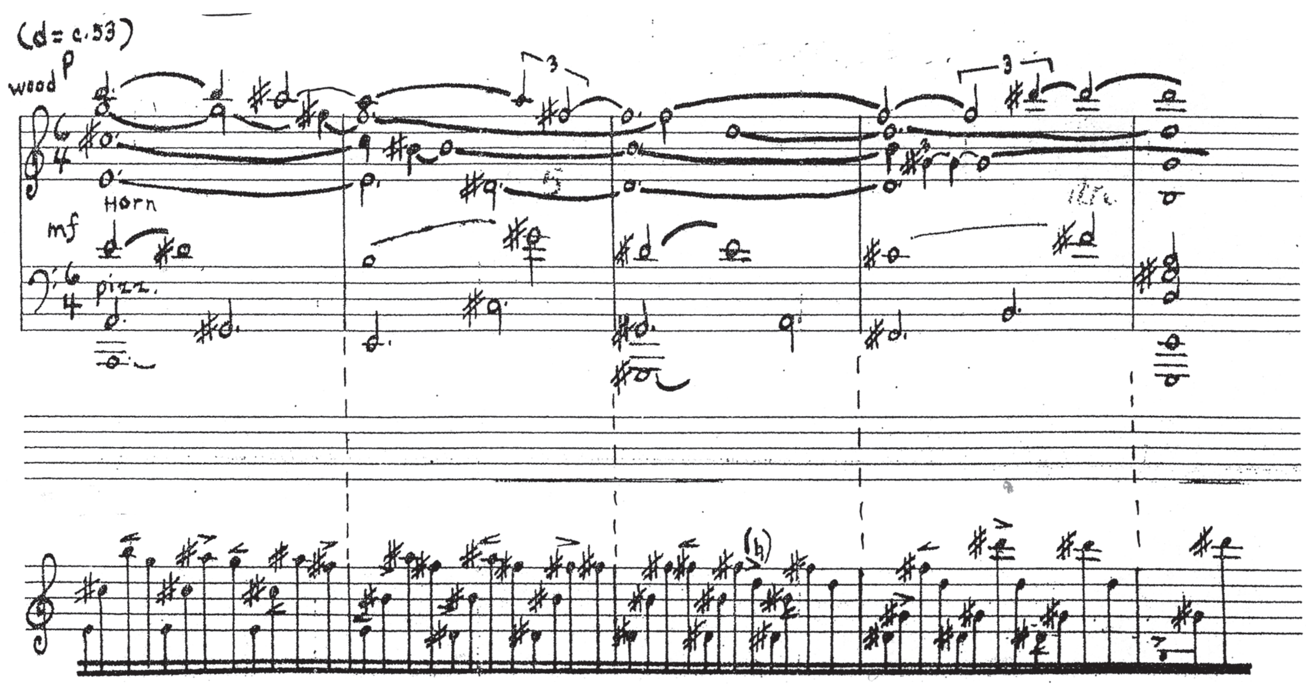
It did not suit me very well, however, at the time, to compose opera. For one thing, I was in the process of finishing *Gilgamesh*; and for another, I knew that a larger symphonic effort had nearly become a necessity for me while I was involved in a continuing research process in music theory of revolutionary nature, which began after the composition of the *Second Symphony* (1970). Fortunately, I was met with flexibility by the Music Department, and received in 1972 a commission for a "large-scale symphonic work." In addition the commission contained some unusual secondary benefits: In the process of composing the symphony, I was given the opportunity to test its sounds with both small instrumental groups and with the full orchestra. This was a generous offer – and, I believe, rather unique? – and I took advantage of it on a couple of occasions by gathering the orchestra for the recording of symphonic fragments, which I could later listen to and modify at my leisure.

The strange thing about this offer was that it coincided with my *need* for it! Normally, I use neither piano nor orchestra for notating my works, but precisely in the period after the *Second Symphony*, the demand for regularly having the resultant sounds controlled by my perception of them was – and is – an absolute necessity. *Why?* It is not because I'm working with "nie erhörte Klänge" [harmonies not previously heard, trans.] *a la* Ligeti's *Atmosphères*; I believe I can hear them in myself without external sonorous confirmations (for example, as in *Luna* and *Iris*). No, the demand for a test in sound emerges from the development toward a totally new, and at the same time very classical, method of composition; i.e., the multi-layered "hierarchic" method.

To call a hierarchic method of composition "classical" is well founded. Baroque, Classical, and Romantic music are, both in a rhythmic and a harmonic sense, veritable apotheoses of hierarchic layers of order, in which micro-rhythms group themselves in bars, which again group themselves in periods, etc. Correspondingly, we observe the same situation in the realm of harmony where the harmonic atoms, three- or four-part harmonies, group themselves in cadence-molecules, which again group themselves in totally stable or modulating organisms, etc. All this is well known, but it is also *acknowledged* to what degree this hierarchic structure facilitates the comprehension of the music because of the many paths in and out of the rhythmic and harmonic fabric of the work?

Arthur Koestler postulates in an article about the nature of hierarchy[1] that whatever forms of life we might encounter on foreign planets, one thing will be certain: they will be built up in hierarchies! It is precisely the absence of hierarchic strata of order that makes much new music so unapproachable and very difficult to memorize. My concern with hierarchic processes of order has increased constantly, partially because my basic melodic source, "the infinity series," is (infinitely) hierarchic and has through the years since its discovery in 1959 forced me to be ever more consistently hierarchic, and partially because I, as a consequence, have conquered myself an increasingly greater insight into, and humility before, the magic possibilities of hierarchy.

The preceding should clarify why the harmonic testing of my symphonic sketches was necessary, precisely because it was not a testing of sound, but a testing of hierarchy: what



Example 1 and (below the empty staff) Example 2 – from "Symphony No. 3"

are the limits for our ability to perceive musical strata, moving in different tempi? For it is self evident that an overburdening of relationships will result in the *greyness* which is so common in new music, even if these relationships were hierarchic, because the very density of sound impulses can overburden the hearing system, as has been shown be recent research.[2]

This problem is related to the problems of the classical polyphony, in which an increase of the number of voices (for example, in a fugue) results in a loss of intelligibility. But the syntax in my "infinity-music" is not only based on separate voices, but also on voice-*hierarchies*, i.e., numerous strata of voice textures, separated from each other into tempo and range relationships.

These experiments with human perception (mine!) were adjuncts of the composition process of a number of works, which all, in one way or another, have emerged from various stages of development of the *Third Symphony*. These earlier works are: *Lila* (1972) for eleven instruments, which is thus the first work based on a coordination of three tempo-strata (in the relationship 1:2:4), each of which encompasses four or five voices, each placed on an individual overtone or undertone "level." The rhythmic disposition progresses in complexity, analogous with the ascendance from fundamental to fifth, third, etc., from periodic rhythm (pulse) in the fundamental voice, over pairs of tones with intervals in "golden section" proportion in the voice at the third level, and so forth (more about this later), as shown in Example 1.

Libra (1973) for tenor, guitar, two mixed choruses and two vibraphones, is the first work which permits the creation of subjective *expression*, by the composer's free choice of accent, colorations, combinations, and alterations within the limits of the objective manifestations of the principles of infinity.

In *Turn* for piano and *Spell* for clarinet, cello and piano (both 1973), the basic organization is the same as in *Lila* and *Libra*, but for the first time the overtone and undertone spectra are distributed in *time*, related to the arpeggiated chord of Baroque music, as, for example, in Bach's *C Major Prelude* (*The Well-Tempered Piano*, No. 1). The advantage is a clarification of the movement of each individual voice, since it is now both characterized by placement within the sound spectrum – such as a fifth of a seventh – and within a "time grid" – such as No. 1, No. 2, and so on, of each pulse module. Surprisingly, even to myself, this creates a bridge backward to works like *Grooving* and *Waves*, in which corresponding grid systems were used with no other motivation than a fascination with the nerve stimulus caused by interference rhythms[3]. Example 2 shows how the irregular progressions of the infinity series become distinct within the different overtone levels simultaneously with the emergence of the interference rhythm as a unified phenomenon. In *Waves* and *Grooving*, this stimulus was the goal; in *Turn* and *Spell* (and then also in passages of the *Third Symphony*), it becomes an adjunct phenomenon to the delineation of the movement of the voices.

Still another work was created out of the "preliminary studies" for the *Third Symphony*, the choral work *Singe die Gärten, mein Herz, die du nicht kennst* for eight-voice mixed chorus and eight instruments (1974). In this work, I utilize the pitch and rhythmic interference which is created by the simultaneity of tempi in the relationship 2:3, combined with the corresponding distance of a fifth between the two basic tone successions.

The development of my work during the past few years has, in an apparently paradoxical manner, progressed toward increasingly greater freedom *concurrently* with the advent of a unified theory of increasingly greater strictness. The paradox can be explained by the fact that the theoretical background forms a texture so dense that it is not only possible but necessary for the composer to choose which tones out of those possible at any given time should be manifested in sound. It is precisely this dense consistency that will assure the succession of the chosen tones will evoke the latent texture in the (sub) consciousness of the listener, in the same way as the tones F and B are sufficient to evoke the effect of the dominant seventh chord.[4] *Singe die Gärten* is almost naïve in its obvious use of the chosen tones in comparison to the possible tones in the basic material. But this simplicity corresponded exactly to my experience of the evocation of the text (not its content!): Singing was suggested, choral singing, i.e., in a *group*, which undeniably favors qualities such as ease of comprehension, singability, lack of complexity, and so on. It later became clear that this choral work was to become the parent of the end of the second movement of the *Symphony* as a *quodlibet*-element, in which its "group soul" simplicity contrasts strongly with the surrounding complex symphonic universe.

This discussion of works that were composed as accompanying studies to my *Third Symphony* might appear cumbersome and belabored; but a closer examination of the *Symphony* will show that most stages and technical conquests are reflected and tested in the series of pieces quoted. And this fact is constantly on my mind when I work on the *Symphony*; I have never previously experienced anything that even approaches such a creative process for a work, a process for which it has been necessary, so to speak, to lay down gangplanks for every step forward. I am also unaware of precedents from music history, but perhaps others can enlighten me in this regard. Along these lines, I would also mention experiments with improvisation on the basis of the principles of infinity which have been started in Copenhagen, Helsinki, Århus, and, of late, at a hign school in Holstebro. Discussing the thought behind this new form for group performance would become an article in itself, so I will limit myself to point out that it does not revolve around "improvisation exercises in Nørgård-style," but that the basic sound texture and its infinite simultaneous succession, on the contrary, serve to further the individual, personal expressiveness within a mutual, multi-dimensional, perceptible frame of reference. (A more comprehensible, but somewhat oversimplified, explanation is that every improviser can periodically *hear* whether he is in phase with his fellow performers – even if he possibly is moving in a completely different basic tempo.)

Though difficult within the limited space of an article, I will attempt to postulate the principles of the creation of this "basic texture" and proceed to explain it in condensed version.

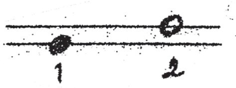
With this attempt the discussion of the Symphony will go beyond the rather uninteresting "descriptive resume," especially because the generating forces behind the work, as mentioned, are manifested in audible, multi-dimensional shapes, and because understanding these forces increases information transmitted during listening.

The principles are bound to the three "elements": melody, rhythm, and harmony. As far as melody is concerned, they are expressed in the infinity-series (hereinafter indicated as ∞ M); for rhythm, in the spectrum from periodicity to total *aperiodicity* (∞ R); and for harmony, in the polarity of the overtone-and-undertone series (∞ H, ɔ: harmony).

I: GENERATION OF THE INFINITY SERIES (∞ M)

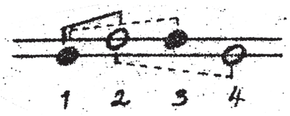
a) Of two tones

Two pitches are given[5],as shown in Ex. 3:



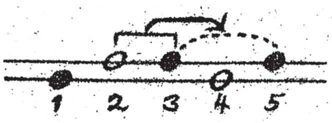
Example 3

The interval between 1 and 2 (equals plus 1) is projected along the time axis in its *original version*, at the position of the third tone, and in *inversion* at the position of the fourth tone:



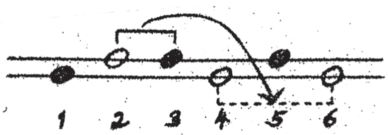
Example 4

Thereafter, the interval between second and third tones (the prime) is correspondingly projected from the position of the third tone to that of the fifth tone:



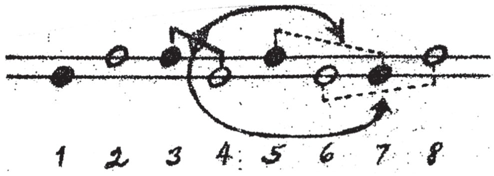
Example 5

– and from the position of the fourth tone to the sixth tone:



Example 6

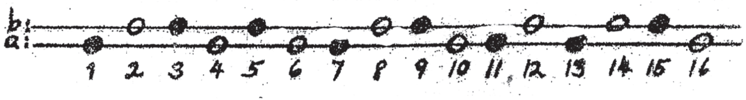
Yet, a (double) step will be considered: The interval between the third and the fourth tone (descending second) is projected in *original* version from the position of the fifth tone to the seventh tone, and in *inversion* from the position of the sixth tone to that of the eighth:



Example 7

Briefly expressed, it can be said *that every interval appearing on the time axis is successively projected from an odd-numbered tone in original version to the position of the next odd-numbered tone, and from an even-numbered tone, and from an even-numbered tone in inversion to the position of the next even-numbered tone*.

Consider the following 16-tone fragment:



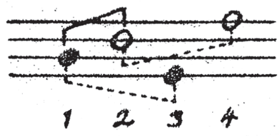
Example 8

It is evident that the procedure results in a hierarchic pattern in which the introductory four-tone figure (A, B, B, A) is *reproduced* in the *original* version in the tones 1, 3, 5, 7, and *inverted* in the tones 2, 4, 6, 8. In addition, it appears in the *original* version in 1, 5, 9, 13 and *inverted* in 2, 6, 12, 16. Two types of pairs (1-2, 3-4, etc.) are observable: ascending (A, B) and descending (B, A).

Also, the succession of pairs reproduces the "A, B, B, A" pattern; the same is true of the groups of four tones, in which there are also two types: "A, B, B, A" and "B, A, A, B". If the first group is called A and the second, B, we find once more the pattern (A, B, B, A). And when raised exponentially in twos it leads to ever more extensive structures, always mutually inverted and always in the A, B, B, A pattern.

b) Of seven tones

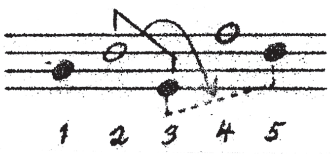
A far more variable ∞ M comes about if the odd-numbered tones are permitted to move in inversion and the even-numbered tones in the original version:



Example 9

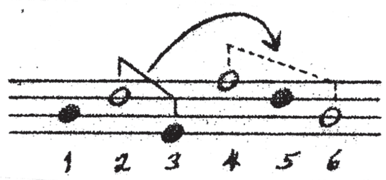
This procedure immediately bursts the two-tone continuum and, consequently, brings into question the scale type on which the process can be monitored. In all works up to *Canon (1970-71)*, the scale was always of twelve tones. But infinitely refined, spiritual, and sensuous proportions of the seven-tone scale (will be explained later in the article) prompted me to work exclusively within the virgin territory of its hierarchic possibilities. In the following, I will, therefore, first show ∞ M's generation in an abstract grid of undefined frequencies (as the reader might want to experiment with by using scale forms of his choice) and thereafter employing the diatonic scale.

After the manifestation of the first four tones (see previous example), follow the projection of the second interval: *two steps down*, which from the third tone is projected in inversion, in other words, *two steps up*, to the fifth tone:



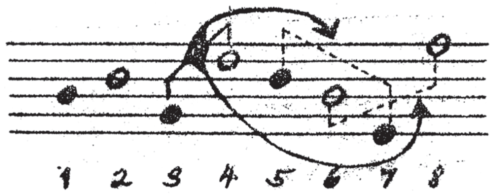
Example 10

- and thereafter, from the fourth tone, projected in original version *two steps down*:



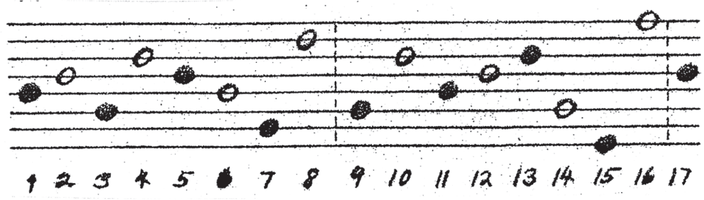
Example 11

Notice that the next interval, between tones 3 and 4, will exceed the previous frequency grid, since the interval *three steps up* has to be projected in inversion from the fifth tone to the seventh tone; correspondingly, it is projected in original version from the sixth to the eighth tone:



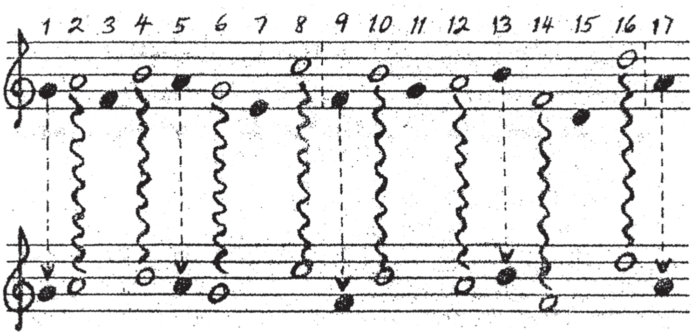
Example 12

It should now be evident that this process is infinite: for every successively projected interval, there result two new tones. Following the stage arrived at above comes the projection of the interval between the fourth and the fifth tone, resulting in tones 9 and 10 and thereafter of the interval between tones 5 and 6, resulting in tones 11 and 12, etc. A section of 17 tones appear as follows:



Example 13

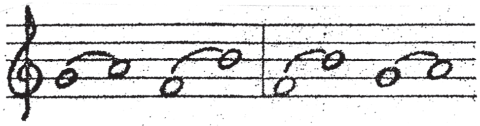
As mentioned, it is possible to choose any scale for the actual realization in tones, but the following example shows the result when worked out in the diatonic scale proceeding from the tone G:



Example 14

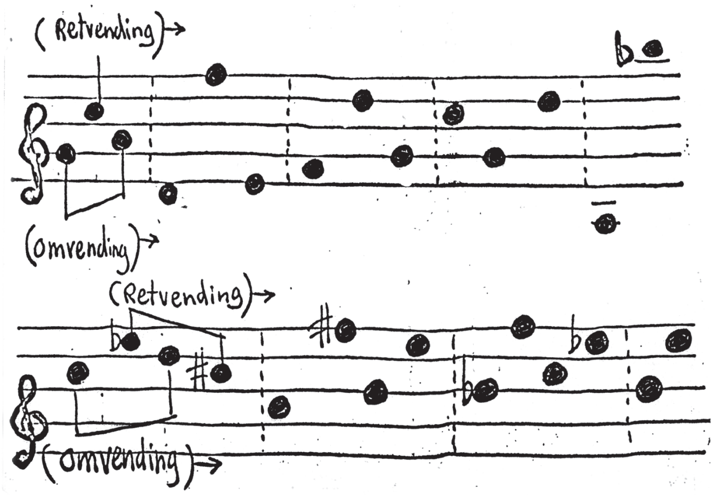
Proceeding from these tones, it is possible to reproduce the original melodic line starting on any of the original tones; in the example, the vertical broken lines (the black tones) are used to show a version of the untransposed original melody as slow, and with the serpentine lines, a version transposed to the second step and twice as slow. The method of observation is in every case: to choose a starting tone, find its continuation while skipping the tones in between, and thereafter continue to preserve the established interval (which is always a power of 2). It can be seen that tones 1 and 17 form the beginning of ∞ M in an untransposed version; it is predictable that tone 33 will be an F. *Every single one of the infinitely many ∞ M tones is the beginning tone of an infinite number of ∞ M in original version and inversion.*

The musically interesting characteristics of ∞ M are, among others, that it is grouped into a series of tone-identities, which return in new successions, in inversion, etc. Alone in the 17-tone fragment, a *cancrizans* shape between tones 1-2, 3-4 and 9-10, 11-12 is observed:



Example 15

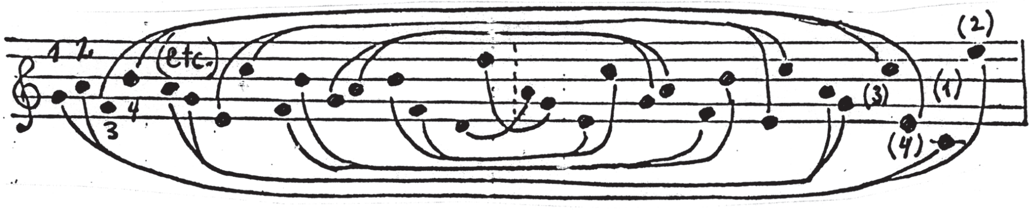
It would go far beyond the limits of this article to carry through a methodical study of ∞ M's phenomenology. The reader who is interested should, from the information given, also be able to generate a larger infinity section for himself and study this in its own context. It will, likewise, be productive to "invent" ∞ M with more than two initial tones and observe how the identity of the initial figure reappears in a new version as if by metamorphosis. The following two examples are based on the chromatic scale and show how, respectively, an identity of predominantly diatonic character results in projections of diatonic characteristics, and a chromatic identity in projections of chromatic characteristics:



Example 16. With Original (Retvending) and Inversion (Omvending).

As a conclusion to this cursory examination of the phenomenology of the infinity series, it can be shown how the two-tone infinity series, far from being a unstructured string of tones, is divided into groups of 16 tones, each centered on an axis formed by the 8th and the 9th tone (Ex. 17):

Example 17. Diatonic infinity series, the first 16 tones.



Example 18. Diatonic infinity series, the first 32 tones.

This symmetric constellation: fifth, prime, prime, prime, prime, fifth, fifth between the tones 1-16, 2-15, 3-14, etc., will appear in every single 16-tone group, and, therefore, also between pairs of tones in the 32-tone groups (Ex. 18), between double pairs in 64-tone groups, and so on.

It is self-evident that here lie latent possibilities for conceptions in large forms: the sonorous "answer" to every statement in the first half of every 16-tone group is found exactly mirrored in the second half, in the reproductions of prime or fifth.

II: GENERATION OF INFINITY RHYTHM SERIES (∞ R)

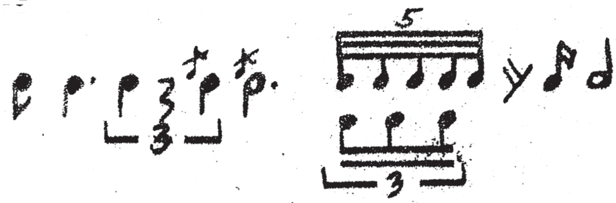
The composition of my *Second Symphony* in 1970 meant the culmination of my ten-year-old study of the chromatic ∞ M. As in the second movement of *Voyage Into The Golden Screen* (1968), but in a musical event of four times the duration, almost all both rhythmic and harmonic invention was subordinated to an intensive concentration on the melodic hierarchies (as indicated in the previous section).

Therefore, all the basic rhythmic characteristics of my *Second Symphony* are "*periodic*", so that no interesting rhythms should distract from the concentration on the melodic movements in their whole, half, fourth, (etc.) tempi.

Following the composition of my *Second Symphony*, I found myself in an enormous rhythmic vacuum – after all these eight notes, quarter notes, and half notes, I had to ask: Why, in fact, should I use other note values? Bach's *Brandenburg Concertos* do fine with the purely periodic rhythmic repertoire! The answer to this emerged from the historical situation: the music since the turn of the century has convincingly demonstrated the effectiveness and expressive power in the aperiodic rhythmic image; particularly, the Frenchmen Debussy, Messiaen, and Boulez permitted unequal note values to succeed each other out of preference. It should here be understood that I do not refer to eighth notes, quarter notes, and half notes as uneven note values (which, indeed, are only powers of the same basic duration), but situations, the common denominator of which lies outside the immediate perception. I mean to indicate that after four eighth notes you can continue to hear eighth notes during a succeeding half note:

Example 19

- but that every individual duration stands by itself, in genuine a-periodicity, in the following example (which is excerpted from Boulez' *Improvisation sur Mallarmé*, typical for this phenomenon:



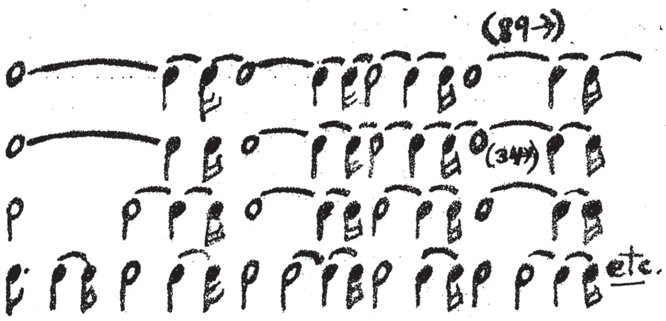
Example 20

The nature of the problem was now clear to me: on one hand, I accepted "successions of irregular durations" which, together with periodicity, form an indispensable rhythmic repertoire. On the other hand, I would not give up those conquests of simultaneous, hierarchic melody layers, which especially had been hard won by the composition of (and not the least during the *listening to*!) the *Second Symphony*. Then, I was faced with what appeared to be contradictory demands: construct a rhythmic spectrum, which unites the hierarchic nature of periodicity:

Example 21

- with the characteristics of a-periodicity (dissimilarity in values from duration to duration)!

I shall not relate the path to this constellation of the factors of the problem, nor shall I relate the stages toward its solution. I shall comment on the following example as 'fait accompli':



Example 22

Translated to numbers, this a-periodic rhythmic chart looks as follows (with continuation):

<table>
<tr><td colspan="8">55</td><td colspan="8">89</td></tr>
<tr><td colspan="4">21</td><td colspan="4">34</td><td colspan="4">55</td><td colspan="4">34</td></tr>
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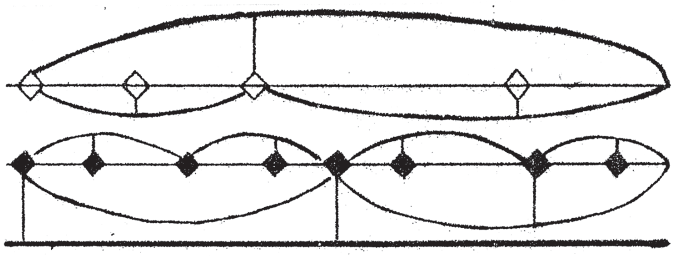
</table>

Example 23

It can be observed that every layer of durations contains all faster layers, while every value simultaneously is a-periodic in relation to the preceding and the succeeding. In addition, every layer is in canonic relation to all other layers. On first sight, it would appear as if the classical metric notation would be able to express these relationships; but since all the numbers are *approximations* to the "golden section" an adequate notation must necessarily be proportional. The traditional notation can then be considered a usable approximation. (That is why *Canon* has been published, notated in two versions, one metric for and one proportional for further studies and truly proportional, i.e., *spontaneous* performance.[6]

In order to express the relationship to the layer-thinking of mensural notation, I have in *Canon* used diamond-shaped notes, which also keeps them distinct from the usual quarter and half notes. The example shows the two rhythmic layers:

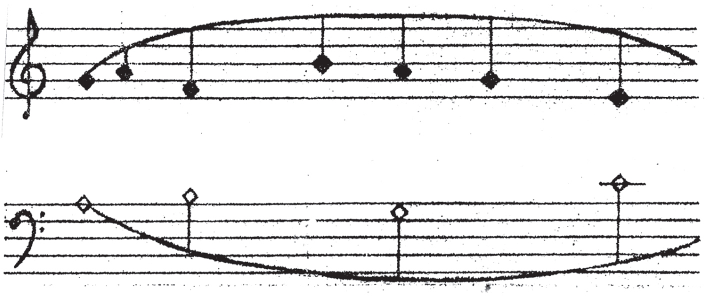


Example 24

The slurs indicate 'golden' proportions between all the tone successions, while the right angles indicate even duration (periodicity) between the enclosed tone groups. A consideration of the relation between the tonal pairs of the slur-groups 1-2 and 3-4 shows that they also are 'golden'. This example of hierarchic a-periodicity must suffice. It is essential that the reader understands my strict distinction between thoughts concerning rhythm, which is in contrast to the coupling during the fifties of the twelve-tone serialism for pitches as well as for durations and attacks (*Structures I* by Boulez). It was liberation for me to acknowledge that, on the contrary, two widely differing structural perspectives should be adopted in regard to the parameters for rhythm and duration (which I, by the way, refuse to call parameters, but rather 'elements' in recognition of this character difference). Where melody moves in time as a series of qualities – since an A is qualitatively something else than a G – rhythm displays an impulse chain of 'empty' durations, which *in se* are without qualitative differences. It is the tones, i.e., the melody, that create the qualitative differences; rhythm is, of itself, only an abstraction of expansion and contraction, and does, therefore, not lay claim to perception of shapes for greater lengths of time. Rhythm distributes the quantity of attention, which is to be given to every melodic tone and is, therefore, especially an expression of sovereign *will*: "To this point and no farther!" Melody is a result of immediately perceptible differences, up-down = fast-slow, in fine proportions within the audible range. The grasped and the degree to which they are being sustained becomes, therefore, the expression of *feeling* these differences.

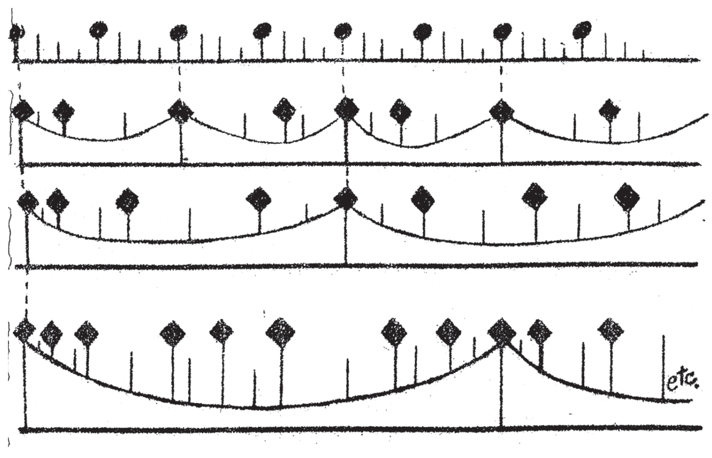
Precisely because of this phenomenon, melody and rhythm form, so to speak, an erotic pair that complement each other, need each other – because their functions are so basically different. The thinking in parameters of the fifties tended to level out differences and was, therefore, immature from a musical viewpoint, while progressive from a developmental viewpoint.

A comparison with ∞ M's coupling to ∞ R will show that the large melodic ups and the identity-permulations are set in *relief* by the hierarchic a-periodicity, which, as far as duration is concerned, lets every tone expend itself, more or less, while durations recede to become purely supporting functions:



Example 25

(∞ M and ∞ R can pair of in steadily new combinations.) Proceeding from the polarity: total periodicity/total aperiodicity, it is possible to establish an (infinite) row of steps from one to the other, all hierarchic:

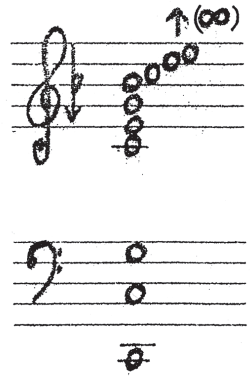


Example 26

As mentioned (and exemplified in example 2), it is possible to distribute this basic proportionality in a time grid for clarification of the movement of every voice functioning within a polyphonic or even hierarchic web.

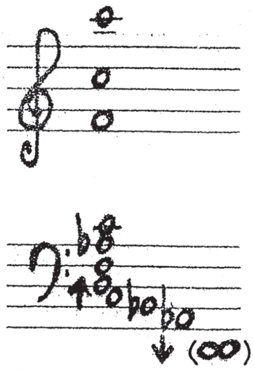
III: THE INFINITY-HARMONY SERIES (∞ H)

The third element of music is harmony. It manifests itself in total balance, when a row of individual tones is in a simple numerical frequency relationship like 1:2:3:4, etc. (the so-called "overtone series"). The aural perception of these series is characterized by the absence of beats. (A clarification of their unequal ranking as 'natural' series would go beyond the scope of this article.) While ∞ H↑ manifests itself as:



Example 27

– the subharmonic (∞ H↓) stack looks as follows:

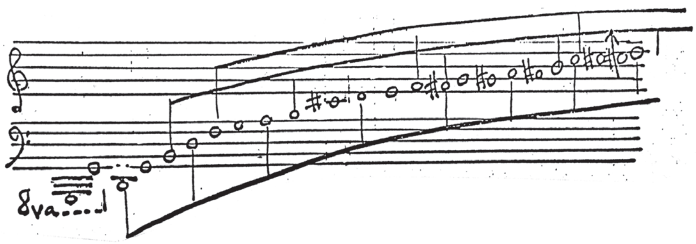


Example 28

– both with C as fundamental.

These two versions (once again, polarized, as in the cases of ∞ M and ∞ R!) are recognized as the major seventh chord and the minor triad (the so-called "subdominant" with an added sixth). It might seem surprising that a 'progressive' composer both employs these harmonies (without irony) and that he uses them within the (to all appearances) just as well known diatonic scale. But an unprejudiced reflection – in as far as possible – will lead to the acknowledgement that these two stacks form two optimal sound grids for perception clarification of the conflict-free encounter of multiple voices. This postulate rests on the concept of a polyphonous (possibly hierarchic) web of tones, intended to be heard by the human ear. Inasmuch as the ear, as is well known, is overtone-sensitive because of its nonlinear physiology, a receptive subject will immediately and un-analytically perceive harmony. Proceeding from this point of reference, the composer is free to utilize the full range of all possible tone combinations from the simplest relationship to the point of the noise polarity.[7]

I might add that at the beginning of *Symphony No. 3*, I also make use of other qualities of ∞ H, particularly its hierarchic content of an infinite number of stratified overtone (respectively, undertone) series:



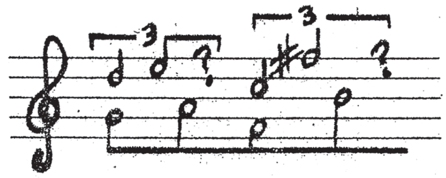
Example 29

I – II – III: COUPLING OF ∞ M, ∞ R, AND ∞ H

The preceding three special studies have shown that the hierarchic structure is mutual to the infinity systems as far as melody, rhythm and harmony are concerned, despite their qualitative differences. It is self evident that the thought of a coupling between them would suggest itself. Let us as a point of departure consider ∞ M's octaves, both in regard to tempo and frequency. An octave expresses a relationship of 2:1; expressed within the perception categories of time and tonal space, a basic octave coupling will look as follows:

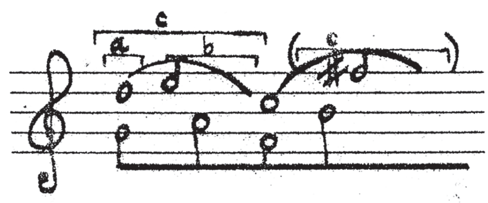
Example 30

This should hardly need further comment. The question of how the relationship of a fifth will express itself durational within ∞ M, however, requires some reflection. The simple manifestation of 3:2 is excluded by the fact that the fifth-version of ∞ M contains the same number of tones as the prime-version (or of its octaves); there is, therefore, no possibility of letting three ∞ M tones correspond to:



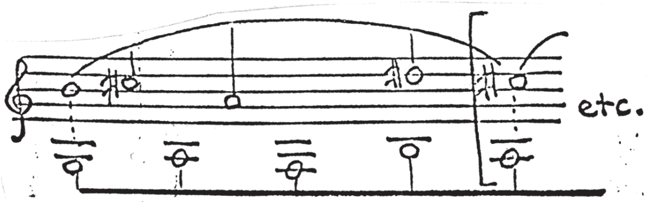
Example 31

Here the "golden section" enters as a necessity: the duration of the fifth becomes inconstant, but harmonically proportioned (corresponding) to the Italian term for the golden section: *divina proporzione*, 'the divine proportion', since it is triune. The lesser of the two lengths is so much smaller than the sum of the two: in other words, three durations are required in order to express one relationship!)



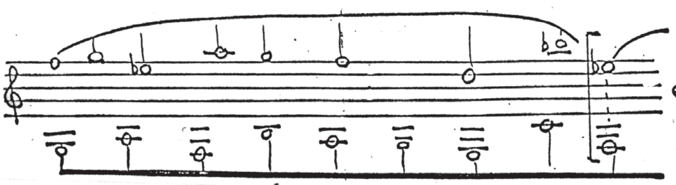
Example 32

With this solution, the road has been paved for all other overtone relationships. The fifth harmonic (the third) becomes in the further refinement of its proportions "double golden" and needs, in other words, four ∞ M tones in order to find expression:



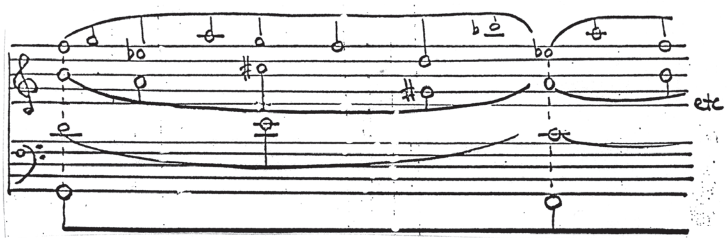
Example 33

The seventh harmonic needs eight ∞ M tones:



Example 34

– and so on. An upward progression in the overtones accompanied by a corresponding octave doubling of the basic tempo will produce a total-harmonic hierarchy:



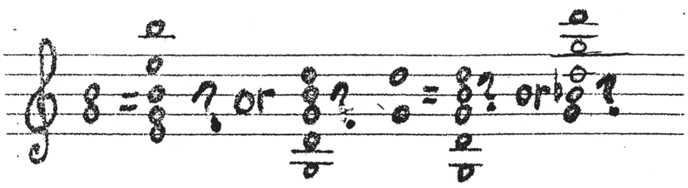
Example 35

It will be seen that this coupling of the three infinity systems leads to genuine polytonality, where in this case the prime periodicity is in "C Major," the golden pair of the fifth in "G Major," the quadgroup third in "E Major," etc. (The reason for quotation marks by the tonalities is explained in the following section.)

IV: SECRETS BEHIND THE DIATONIC SCALE

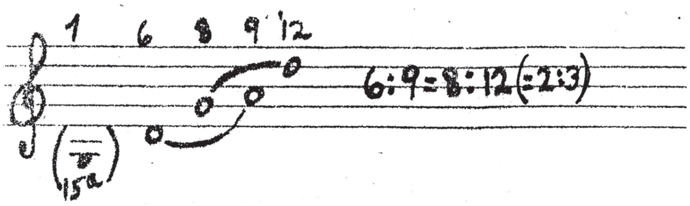
"Why are there seven tones in the diatonic scale?" This question is usually answered on purely historic grounds as planetary symbolism, etc. In other words, with references which do not exactly point to any deep inexorable elementary order which permitted these seven, and only these. There is, however, an explanation which is all but unknown, at any rate overlooked, which goes all the way back to the late Pythagoreans. Albert von Thimus, in his masterpiece of 1876, *Die harminokale Symbolik des Alterthums*[8], was the first to interpret this formula, which had been handed down by Jamblichus/Nicomachus.

I shall come directly to the main point. Acceptance of harmonic and sub-harmonic harmony means that three tones are required for definition of a tonic. For example, G-B could be interpreted both with B as tonic in a B-G-E chord, i.e., sub-harmonically, and with G as tonic in a B-G-D harmony, as the overtone harmony on G. And for the tones G-D, we are lacking the determination of whether it is a B or a B-flat that is understood, which means that it is uncertain whether G is the tonic in a harmonic or D in a sub-harmonic chord:



Example 36

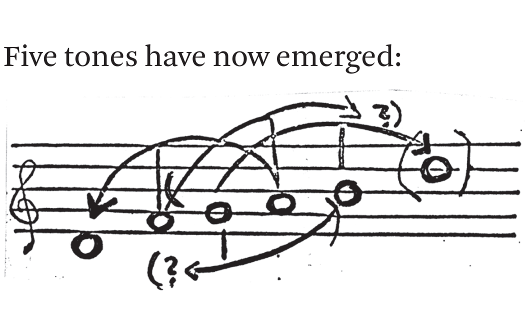
Three tones are, in other words, required! Emanating from the proportions 6:8:9:12 ("Tetraktys"), we find two perfect fifths within the overtone series:



Example 37

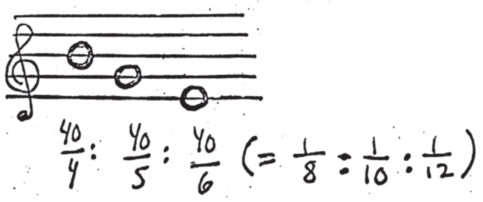
If we make G the tonic for an overtone harmony ("G Major") and A the tonic for an undertone harmony ("D Minor"), we have for "G Major" 8:10:12 and for "D Minor" (starting from A) 1/8:1/10:1/12. When we place 1/8 =9 (A) and 1/12 = 6 (D), this can be rewritten as 72/8:72/12, through which we find that 1/10 will be called 72/10; in other words, 7.2. This 7.2 is the third in the sub-harmonic harmony, which is called "minor" and contains the tones A-F-D.

Five tones have now emerged:



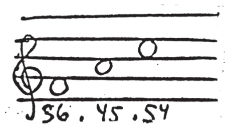
Example 38

– but we perceive two axes of thirds, two two-tone harmonies, one on B-G missing the fifth below, E, and one on F-A, lacking the fifth above, C. B = 10 is rewritten as 40/4; G = 8 as 40/5, whereby E becomes 40/6 = 6 2/3 :



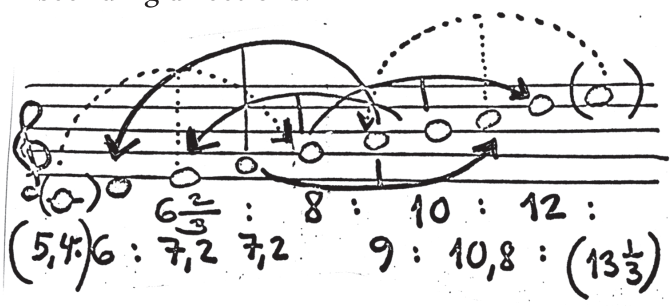
Example 39

F = 7.2 is rewritten as 36/5; A = 9 as 45/4, whereby C becomes 54/5 = 10.8



Example 40

By this process, a complete seven-tone, harmonic-subharmonic system of relationships has emerged, emanating from the two D's that frame the system; all major thirds are now projected into their ascending and descending directions:

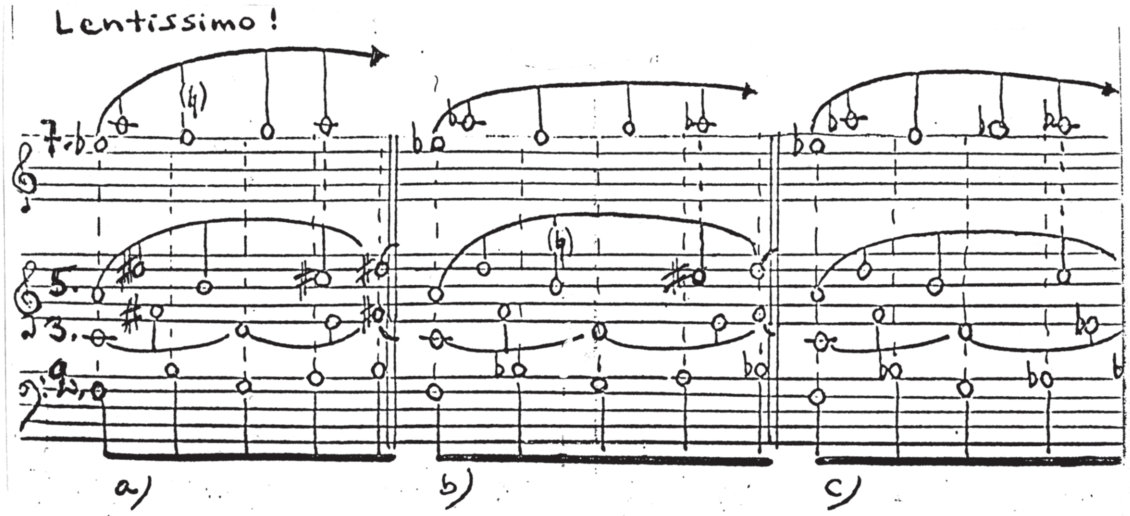


Example 41

Additionally, this produces two 'impure' overtone and undertone harmonies: (C-E-G = **5.4** : 6 2/3 : 8 instead of **5** 1/3, etc., and E-C-A = **13** 1/3 : 10.8 : 9 instead of **13** 1/2, etc.). Further, the two corresponding church modes, Ionian and Aeolian, were given derogatory descriptions, 'modus lascivus' and 'modus peregrinus' – 'the lascivious' and 'the alien' – which is perhaps not without some connection to this (deeply hidden) impurity behind their generation.

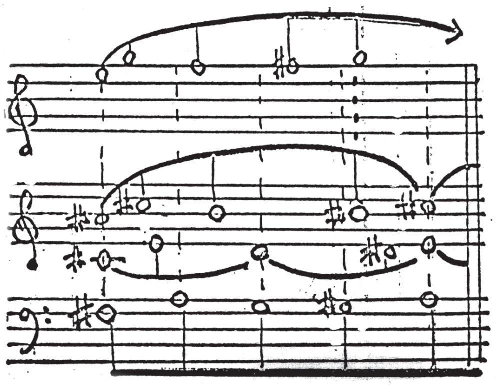
The discovery that there was a comprehensible, high-level system behind the well-known seven-tone scale was, for me, something of a revelation with far-reaching consequences. In the first place, thinking with "seven modifiable tones" signified a considerable liberation and expansion in relationship to the "twelve fixed tones." With a deeper familiarity with the synthesis (I-II-III) of the infinity systems, it will become apparent that precisely by virtue of the *durational* displacements between the overtones cause by the "golden proportion" with a single tonic will have far-reaching fundamental results for the harmony within a larger time space. An example will serve to clarify the latter. Playing through an ∞ fragment in which a pair of tonics, with their spectrum, one by one are lowered should clarify the enormous possibilities of the seven-tone scale in connection with the ∞ hierarchies (Ex. 42 should be played slowly with careful registration of the differences):

A comparison with the same fragment on the basis of the chromatic twelve-tone scale will show that in this case it would have one



Example 42

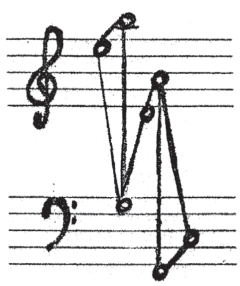
version and one version only, as the twelve-tone space by definition is not capable of chromatic modification:



Example 43

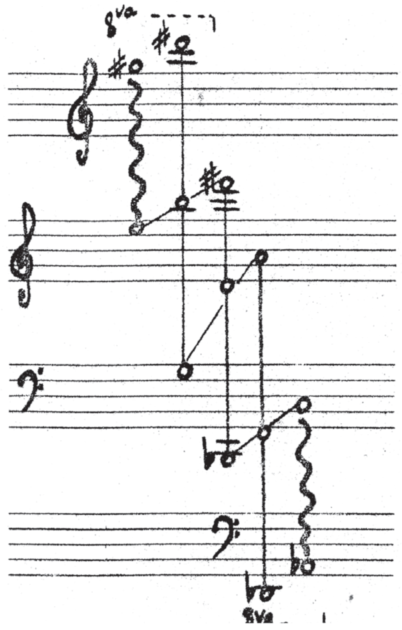
(If anyone has a special preference for this example, it should be recalled that this result is also possible with a seven-tone scale; it is only required that the fundamental tones are modified as follows: F-sharp, B-double flat, G, A-flat, B-double flat).

Understanding of the true nature of diatonicism and its wisdom led to still one more conclusion: my expansion of the seven tones to include the five hidden ones led to a tonal system with which I have experimented with promising results. The prime characteristic of the seven-tone tuning system proposed by Thimus is that it, in contradistinction to all others of the more-than-a-dozen proposed Western tuning systems, is based upon two pure fifths and not three, but it applies, on the other hand, the natural third, both ascending and descending:



Example 44. The intervals connected with lines are pure.

Based upon this comprehension, it was possible for me to construct a twelve-tone space in which only the interval D to A-flat was well-tempered. It can be seen that it is the natural major third which projects itself respectively upwards and downwards and becomes the foundation of the new fifth-space:



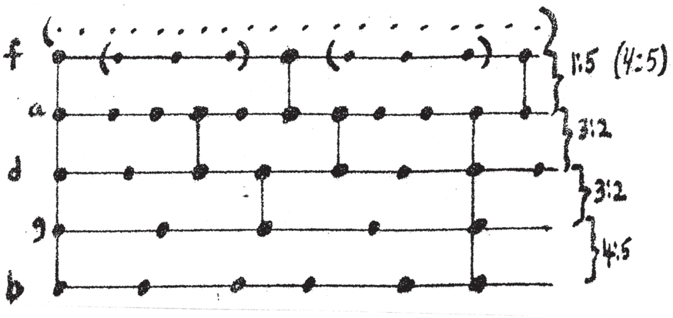
Example 45

(The serpentine line indicates that G-sharp and A-flat are, respectively, too high and too low; as mentioned, there is a well-tempered tone between the correct G-sharp divided by 8 1/3 and A-flat divided by 8 16/25.) These tones are easily manifested in the tunings of the piano.

This tuning is naturally of optimal use for works like *Turn* and *Spell*, which are in their composition based upon the axis G:A; but I must confess that I also find pleasure in classical works played in this tuning. The beats that are created between, for example, C and G are tolerable and even attractive (in the same way that people from Bali find the beats attractive in pairs of metallophones that are purposely tuned in conflict). It is essential and fascinating that with this system of tuning, the movement from key to key becomes a movement of perception, rich in perspectives, inasmuch as every individual key has beat characteristics of its very own. (It is also liberating to break the sterile well-tempered tunings...!).

V: DIATONICISM AS THE BASIS FOR RHYTHMIC PROPORTION

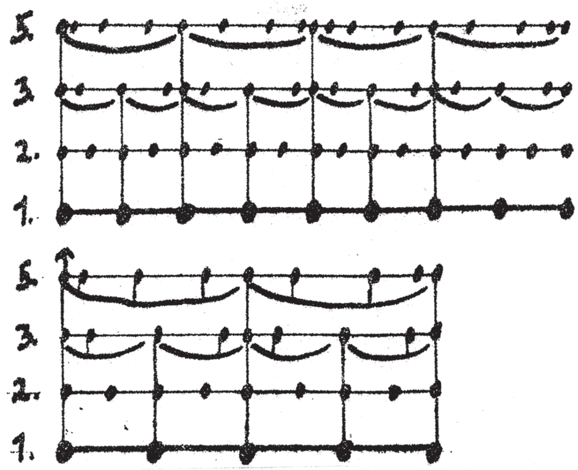
The intervallic relationships mentioned above cast an entirely new light over the phenomenon of polyrhythm when transferred to the rhythmic plane. The latter occurs when frequencies are expressed as tempo-relationships.



Example 46

What is new here is naturally not the differences in tempo in themselves, but the concept that every tempo can form the foundation for a complete octave hierarchy. Therefore, it has been indicated in parentheses by F how 1:5 can be substituted by 4:5, 8:5, etc.

The question is how these latent tempo hierarchies can be perceived as something other than ordinary polyrhythms; that is, how can the listener single out only two of these tempi simultaneously during the full experience of each of them as a complete hierarchy? – For example, 3:2 (Ex. 47).



Example 47

This example reminds us of the synthesis of the ∞ systems, which, however, in all the earlier explanations concern only one tempo and its spectrum of octaves, fifths, thirds, etc. The solution lies, however, not only in coupling two such hierarchies together, for example, in the tempo relationship 3:2. Naturally, "it can be done," but the result would be yet another example of the many half or totally cacophonic works of our century. We will have exceeded the ability of the ear to analyze:

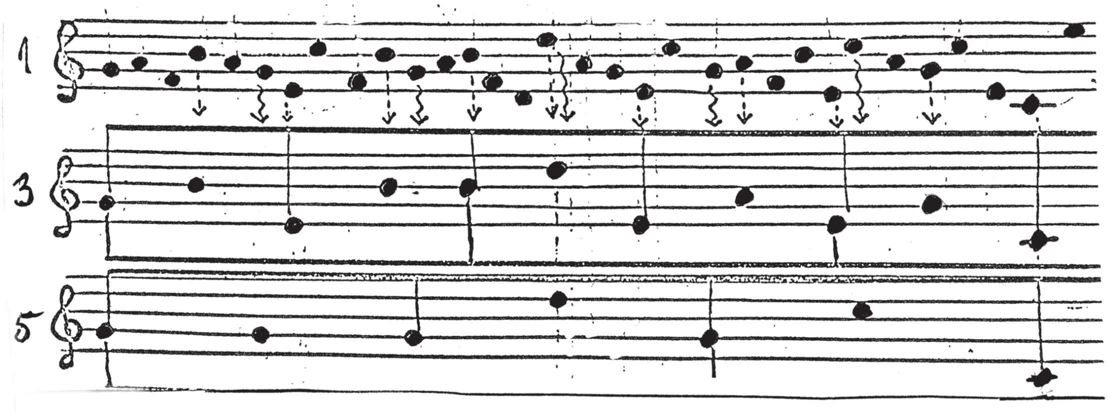
Example 48

It is immediately apparent that this coupling of an ∞ M in tempo three with another in tempo two from a structural viewpoint is quite arbitrary, and at the key points only results in "accidental" encounters, "noise." The vision of a tone hierarchy which is multidimensional in regard to tempo would have remained an Utopia, had the fundamental two-tone generated ∞ M not possessed a fantastic quality, which I first discovered in the summer of 1974 – in other words, a quality that remained undiscovered during 14 years of work with the infinity series and the exploration of its manifold relationships. Briefly formulated, this quality manifests itself as follows:

The fundamental ∞ M is not only hierarchic in powers of 2 – (in other words, 4, 8, etc., as shown in section I) – but in powers of all numerical relationships!

Let us, for example, look at the frequently quoted introductory fragment (Ex. 14) and expand it by doubling (Ex. 18), in this new perspective (see Ex. 49).

By using ∞ M's tones in series of every third one (1,4,7,10, etc.; indicated by a broken-line arrow), two every fifth one (1,6,11,16, etc., snake-line arrow) two new melodies appear, which both possess ∞ M's fundamental quality: they are *hierarchic*. The criteria are (again) that every second tone forms the inversion



Example 49

and every fourth tone the original. Thus, the inversion of the derivative of the three series:

Example 50

and the derivative of the five series:

Example 51

We observe that both of these take shape from the unequal number of tones of the melodies. The reader is encouraged to try other derivatives.

This creates the possibility of realizing the notion sketched at the beginning of the section: that the melody of every tempo can become the foundation for a totally developed hierarchy. The precondition is merely to find the basic numbers for the "secret" diatonic scale which permits all other proportions to be whole numbers.

Since both thirds and fifths are part of the scale ($6\ ^{2/3}$, $7\ ^{1/5}$, and $10\ ^{4/5}$), it is therefore necessary first to multiply all the numbers with three times five (equals 15) in order to create whole numbers. Thereafter it must be recalled that the numbers with which we are working are frequencies, while what we seek are "wavelengths"[9]. By way of illustration, G:8 and A:9 are respectively, 9:8; in other words, inversely proportional. In the example with the derivatives of series in three's and five's, we observe that "every third one" results in a frequency of five tones for every third in the series of five's. High frequencies have small wavelengths and vice versa; we can, therefore, chart the frequencies of the seven-tone scale once more, multiply them by 15 (to find the whole frequency numbers) and thereafter take their inverse proportions, "the wavelengths":

grundfrekvenser	6	$6\frac{2}{3}$	$7\frac{1}{5}$	8	9	10	$10\frac{4}{5}$	12
×15	90	100	108	120	135	150	162	180
bølgelængder	180	162	150	135	120	108	100	90

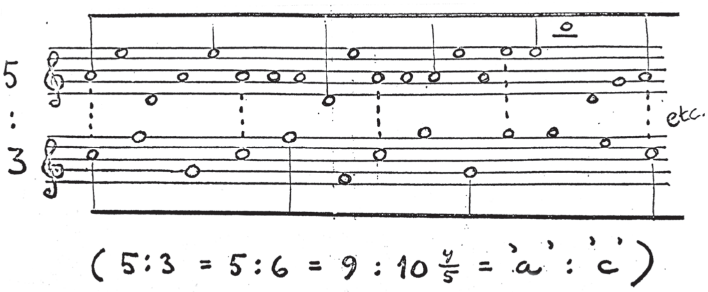
Example 52

These numbers for wavelengths are thereafter divided with as many powers of 2 as possible, resulting in the following valid number series:

$\frac{180}{4}$	$\frac{162}{2}$	$\frac{150}{2}$	135	$\frac{120}{8}$	$\frac{108}{4}$	$\frac{100}{4}$	$\left(\frac{90}{2}\right)$
45	81	75	135	15	27	25	(45)
d	e	f	g	a	h	c	(d¹)

Example 53

This means in practice: If it is desired to let a melody move in relationship to another melody in the tempo 5:3 (for example, A:C), every fifteenth tone should be chosen from the ∞ M, starting on the transposition A, and every 25th on starting tone C. Both melodies will be hierarchic, each individually, and their points of coincidence will always be in harmony. In this case, they will always meet on a minor third:



Example 54

To illuminate any further the dizzying possibilities that emerge from this process would require a special study, and it must be left to the reader, who, in his mind, is capable independently to combine the information given in this article.

CONCLUSION: AND THE SYMPHONY?

It is clear for me that I have related very little of the Symphony in regard to its form and external shape, but quite extensively about its inner structure, which is best compared to a living organism. The formal progress of the Symphony is always identifiable in terms of the state of its harmonic development by an "analytical incision" made at any given time in the course of its evolution. Therefore, I now face the challenge of exploring the possibilities within this infinite sound space in all its infinite chromatic modifications and examining them from the vantage point of different hierarchic strata. At the time of this writing, the second movement is materializing in my mind, the movement in which all the expressive encounters of the tempo worlds shall reveal themselves. The first movement develops as a result of the fundamental cyclic orders subject to the 16- hierarchy of the basic series, subdivided in the 12 tonalities of the circle of fifths. A cycle of peculiar character emerged, which possessed striking similarities to the twelve signs of the Zodiac of ancient astrology.

This discovery made a strong and positive impression on me, inasmuch as astrology is, according to Ernst Cassirer, "considered from a purely formal viewpoint one of the grandest attempts at a systematic, constructive view of the world which has ever been dared by the human spirit."[10]

And it is precisely this "formal viewpoint" which is recovered here in the sonorous time-universe created directly from *ratio*.

There will be two movements.

■

Notes

1 Arthur Koestler, "beyond Atomism and Holism – The Concept of the Holon" in *Beyond Reductionism* (The Alpbach Symposium), (London: Hutchinson, 1969), p. 196.

2 Fritz Winckel, *Phänomene des musikalischen Hörens*, (Berlin: Max Hesses Verlag, 1960), p. 90.

3 "Interference-tension" in music is a terminology I used for the first time in a series of lectures in California in 1970. The concept is divided into micro-, moment-, and macro-interference: micro-interference forms the well-known beats under the limits of the grasp of the analytical ability, in which two tones are so close to each other that they produce a third between them, with the difference in frequencies as the rhythm of the beat; moment-interference is used in *Grooving* and *Waves*. It consists of the repetitions of two or more tones, each with its individual dynamic waves, or two individual tone patterns, the conflict of which creates a confusion of the ability to analyze.

Finally, macro-interference should be understood as melodic lines with mutually interfering periods (for instance, as in the *Second Symphony*), in which the memory-dependent ability to conceive of musical shapes is brought into a corresponding conflict.

4 The phenomenon is analogous to the subjective production of tonic by "the creative

ear," if only two harmonic tones sound objectively.

5 The division into black and white noteheads is merely a convenience: the black stands for odd numbered, white for even numbered.

6 Wilhelm Hansen Edition, Copenhagen, 1974

7 It follows that harmony with a certain justification could be considered an expression of the encounter situation and could become the basis for measurement, *thinking*.

8 Alb. Freiherr von Thimus, *Die harmonikale Symbolik des Alterthums*, reprinted 1972 by George Olms Verlag, New York and Hildesheim.

9 Frequency number: The highest frequency is ∞ M in its original form as it defines time itself, because ∞ M is *all tones* in an infinite time development. The high-frequency number is 9, the lower is 8 (respectively, 'A' and 'G').

Wavelengths: Higher frequencies snatch smaller pieces of ∞ M than lower frequencies in an inversely proportional relationship. The wavelength of 9 ('A') is therefore 8, while 8 ('G') is 9, etc.

10 Ernst Cassirer, *Begriffsformen des mythischen Denkens*, 1922.

■

Translated by L.K. Christensen, 1975 from the Danish original version (*Inde I en symfoni*, 1974)

Printed in the journal Numus West, vol 11, no. 2, spring 1975.