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# A Model for Milk Production, Transportation and Storing at Multiple Plants and Under Fluctuating Supply and Demand\*)

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## *Summary*

Consider a set of plants with given capacities for milk production and storing and for transportation between these places. For a given supply of raw milk (whole milk) and a demand for milk products, how should a short term operation be performed so that the plants are optimally coordinated?

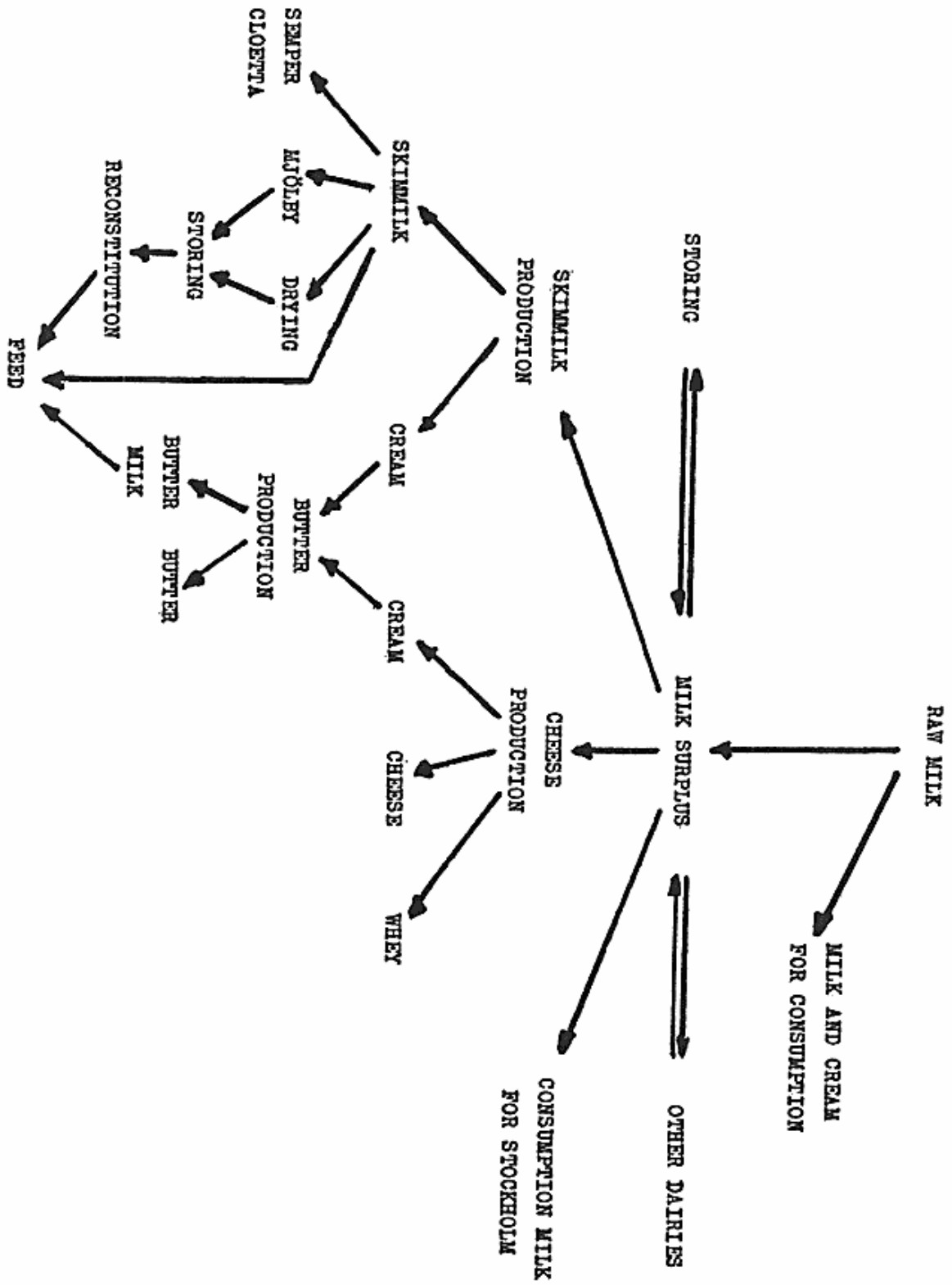
In this study the problem is formulated as a linear programming model, where the total operating costs have to be minimized, subject to a set of constraints for production, transportation, and storing. The model has been applied to a multi-plant firm in central Sweden, where thirteen dairies were coordinated into weekly plans. The result did show that the costs for production, transportation, and storing must be considered in order to obtain an efficient plan.

## 1. *Introduction*

The »Mjölcentralen« (Milk Center) in Stockholm, Sweden, is a cooperation composed of a large set of farmers and dairies in central Sweden. Its long term objective may be seen as to maximize profit subject to a given milk supply. In order to do so, milk production has been divided between separate dairies, each with a specific production program so as to take advantage of economies of scale for each product. As milk supply is regionally dispersed, the dairies have been spread out over the production area in order to lower the costs of transportation.

We will now deal with short term planning of milk surplus. In the short run the plant capacities will be assumed fixed. If we consider a very short

\*) This paper is based on work performed while the author was at the Department of Business Administration at the University of Lund, Sweden.



time period, week for example, milk product demand will also become fixed. This may result in a short term objective of minimizing the costs in production, transportation and storing subject to a given supply and demand to given capacities and to given storage sizes. For the »Mjölkcen-  
tralen« the milk supply may fluctuate over the days of the week, so that one problem will be where to store the milk surplus for some days in order to use production and transportation capacities efficiently? Even for a small number of dairies this problem may become complicated, why a well-organized plan could be economical.

## 2. Milk Production

Let us consider the specific milk production flow of the »Mjölkcen-  
tralen«. The milk is produced at farms and transported to preassigned dairies, mainly for administrative reasons.

When the raw milk has reached the various dairies, a certain part of it will be consumed immediately in terms of milk and cream. The rest of the raw milk will be called the milk *surplus*<sup>1</sup>). This surplus may be used for producing skim milk, dried milk, cheese, butter, and butter-milk. Our problem will be how to schedule production, transportation, and storing of this surplus at the lowest short term costs.

Let us assume a planning period of one week. The daily milk surplus at a dairy can then be used in the way shown in figure 1.

The daily milk surplus may be reduced by a transfer of raw milk to other dairies or by storing. It may be increased by receiving raw milk from other dairies or by reducing the existing storage. The rest of the surplus may be divided between cheese and skim milk production.

The cheese production yields cheese, whey, and cream. The cream is obtained when the raw milk is separated for cheese-making (with a lower fat content than that of raw milk).

The skim milk production results in skim milk and cream. The skim milk will then be used for consumption near the dairies (i. e. as feed), for consumption in certain outside production plants, and for drying<sup>2</sup>). The cream received from both cheese and skim milk production will be used for butter production. From this we will get butter and butter-milk. The butter is sold and the butter-milk is used for feed somewhat replacing skim milk.

1) If there is a milk shortage in a dairy, it will be interpreted as a negative surplus.

2) Skim milk drying is performed by the Mjölkcen-  
tralen in order to be reconstituted during other seasons of the year. The reconstituted skim milk will then be used for feed.

The technical coefficients in this milk production flow may differ between dairies and between seasons. However, we will assume here that from 1.111 kilogram of milk we can obtain 1 kilogram of skim milk and 0.111 kilogram of cream. But 1.026 kilogram of milk can alternatively yield 1 kilogram of milk for making cheese and 0.026 kilograms of cream. Then, from one kilogram of cream we will get 0.5 kilograms of butter and 0.5 kilograms of butter-milk.

### 3. Model Definitions

For the milk problem in this study, the short term planning objective is to minimize operating costs subject to given demand, supply and capacity constraints. One way to carry out this kind of plan is to construct a mathematical model, where an objective function will be minimized subject to a set of constraints.

Let us consider the planning period of one week divided into four sections, 1) Sunday, 2) Monday, 3) Tuesday to Friday, and 4) Saturday. The four days Tuesday, Wednesday, Thursday and Friday can be regarded as one section, since the milk surplus, production demand and capacities are approximately the same. Denote the time sections by  $t$  ( $t = 1, 2, 3, 4$ ) and the dairies by  $i$  ( $i = 1, 2, 3, \dots, I$ ). Then for each dairy ( $i$ ) and this section ( $t$ ) there are variable quantities to be determined in production, transportation and storing.

The five primary production variables are the quantities of 1) skim milk for drying, 2) skim milk for feed, 3) skim milk to be reconstituted, 4) milk for making cheese under normal costs and 5) the same milk under increased costs (in overtime work). Let us call these variables  $X_{i1}^t$ ,  $X_{i2}^t$ ,  $X_{i3}^t$ ,  $X_{i4}^t$  and  $X_{i5}^t$  respectively.

The variable transportation quantities are 1)  $Y_{ij1}^t$  and  $Y_{ij2}^t$  for raw milk to other dairies under normal costs (by truck) and under increased costs (by train), 2)  $V_{i1}^t$ ,  $V_{i2}^t$ , and  $V_{i3}^t$  for skim milk to three external production sites, and 3)  $W_{ij}^t$  for cream transportation to certain butter production dairies.

Finally there is one type of storage variable, i. e. the inflow of raw milk ( $S_i^t$ ). This inflow at time section  $t$  will then become the outflow from this storage at time section  $(t+1)$ .

For each of these variable activities we are interested in the short-term operating costs. Given production, transportation and storage capacities these costs can be approximated as linear of the quantities, except in the case of making cheese and raw milk transportation. Cheese-making at these dairies can be increased over normal capacities through overtime work but

this involves considerably higher production cost. In the same way raw milk can be transported by rail in larger quantities than the existing truck capacities. However, this must be done to a higher (variable) cost.

Formulate these variable costs as:

- a)  $C_{i1}^t, \dots, C_{i6}^t$ , for one unit of the production variables  $X_{i1}^t \dots X_{i6}^t$ ;
- b)  $C_{ij1}^{2t}, C_{ij2}^{2t}$ , for one unit of the transportation variables  $Y_{ij1}^t$  and  $Y_{ij2}^t$ ;
- c)  $C_{i1}^{3t}, C_{i2}^{3t}$  and  $C_{i3}^{3t}$  for one unit of the transportation variables  $V_{i1}^t, V_{i2}^t$  and  $V_{i3}^t$ ;
- d)  $C_{ij}^t$  for one unit of the transportation variable  $W_{ij}^t$ ; and
- e)  $C_i^t$  for one unit of the storage variable  $S_i^t$ .

In our short term planning the capacities, supply and demand can be assumed to be given. For each time section,  $t$ , and each dairy,  $i$ , the following maximum quantities are assumed:

- $B_{i1}^t$  = skim milk for drying
- $B_{i3}^t$  = reconstituted skim milk
- $B_{i4}^t$  = cheese-making to normal costs
- $B_{i6}^t$  = - - - overtime costs
- $M_{ij1}^t$  = raw milk transportation to dairy  $j$  under normal costs
- $M_{ij2}^t$  = raw milk transportation to dairy  $j$  under higher costs
- $L_i^t$  = raw milk stored for time section  $(t+1)$

Moreover for each  $i$  and  $t$  determine

- $N_i^t$  = the quantity of milk surplus
- $D_{i1}^t$  = the quantity of milk (skim milk or butter milk) to be used as feed in that area.

For each of the three outside production sites ( $k = 1, 2, 3$ ), determine the maximum and minimum amount of skim milk they will accept per section:

- $\overline{G}_k^t$  = the maximum quantity
- $\underline{G}_k^t$  = the minimum quantity

Finally for the whole planning period, the following fixed demand quantities are given:

- $D_{2k}$  = skim milk in demand at the outside site  $k$   
 $D_3$  = skim milk in demand for drying  
 $D_4$  = skim milk in demand from reconstitution  
 $D_5$  = milk in demand for making cheese

and the stored quantities at the start and the end of the planning period are given as

- $S_i^0$  = the quantities of raw milk stored at dairy  $i$  when the planning period begins  
 $S_i^4$  = the quantities of raw milk stored at dairy  $i$  when the planning period ends

Observe that the total quantities of milk surplus plus the stored quantities ( $S_i^0$ ) at the beginning of the week, must correspond to the total quantities of raw milk for production plus the final inventories ( $S_i^4$ ).

Such a control must be done before the model can be used.

#### 4. *The Linear Programming Model.*

The planning problem as stated was to minimize the short-term operating costs, subject to the fact that the raw milk surplus is used in production in order to satisfy a given consumption of different milk products. As the production, transportation and storing costs were linear (or piecewise linear) of size, the problem can be formulated into the following linear programming model.

The objective function to be minimized is the sum of the operating costs, i. e.

$$Z = \sum_{i=1}^I \sum_{t=1}^4 \{ \sum_{j=1}^I C_{ij}^{1t} X_{ij}^t + \sum_{j=1}^I \sum_{k=1}^2 C_{ijk}^{2t} Y_{ijk}^t + \sum_{k=1}^3 C_{ik}^{3t} V_{ik}^t + \sum_{j=1}^I C_{ij}^{4t} W_{ij}^t + C_{i1}^{5t} S_{i1}^t \}; \quad (1)$$

The constraints to be considered are of different types. First, the raw milk used in production (i. e. the produced quantities multiplied by their respective technical coefficient) must equal the supply for each dairy and time period. Raw milk stored from the preceding time period will then increase the supply, and the milk not used will be stored until the next period:

$$N_i^t = 1.111 (X_{i1}^t + X_{i2}^t) + 1.026 (X_{i4}^t + X_{i6}^t) + \sum_{j=1}^I (Y_{ij1}^t + Y_{ij2}^t - Y_{ji1}^t - Y_{ji2}^t) + 1.111 (V_{i1}^t + V_{i2}^t + V_{i3}^t) + S_i^t - S_i^{t-1}; \quad (i = 1, 2, \dots, I) \quad (t = 1, 2, 3, 4) \quad (2)$$

Second, the production variables have upper or lower bounds, as:

$$X_{ij}^t \leq B_{ij}^t; \quad (3)$$

$$\begin{aligned} & (i = 1, 2, \dots, I) \\ & (j = 1, 3, 4, 5) \\ & (t = 1, 2, 3, 4) \end{aligned}$$

Third, the transportation quantities between the dairies are restricted as:

$$Y_{ij1}^t \leq M_{ij1}^t \quad (4)$$

$$Y_{ij2}^t \leq M_{ij2}^t \quad (5)$$

$$\begin{aligned} & (i = 1, 2, \dots, I) \\ & (j = 1, 2, \dots, J) \\ & (t = 1, 2, 3, 4) \\ & (i \neq j) \end{aligned}$$

The total amount of skim milk transferred to each outside production site is bounded upwards and downwards:

$$\sum_{i=1}^I V_{ik}^t \leq \bar{G}_k^t; \quad (6)$$

$$\sum_{i=1}^I V_{ik}^t \geq \underline{G}_k^t; \quad (7)$$

$$(k = 1, 2, 3)$$

$$(t = 1, 2, 3, 4)$$

Fourth, the stored quantities of raw milk are bounded by the given capacities:

$$S_{i1}^t \leq L_i^t; \quad (8)$$

$$(i = 1, 2, \dots, I) \quad (t = 1, 2, 3, 4)$$

Fifth, the cream transported from each dairy to the butter-production dairies must be equal to the cream received in production, which can be obtained by multiplying the production quantities by the corresponding technical coefficients:

$$0 = -\sum_{i=1}^I W_{ij}^t + 0.026 (X_{i4}^t + X_{i5}^t) + 0.111 (X_{i1}^t + X_{i2}^t + V_{i1}^t + V_{i2}^t + V_{i3}^t); \quad (i = 1, 2, \dots, I) \quad (t = 1, 2, 3, 4) \quad (10)$$

Sixth, the demand for milk to be used for feed has to be filled either from reconstitution, skim milk production, or (at the butter-production dairies only) from butter-milk. The quantities of butter-milk are equal to the cream transported to such a dairy, multiplied by the corresponding technical coefficient (here 0.5):

$$D_{i1}^t = X_{i2}^t + X_{i3}^t + 0.5 \cdot \sum_{j=1}^I W_{ji}^t ; \quad (11)$$

$$(i = 1, 2, \dots, I)$$

$$(t = 1, 2, 3, 4)$$

Seventh, the rest of the supply must be equal to the demand. The total supply of skim milk to each outside production site  $k$  must then be:

$$D_{2k} = \sum_{i=1}^I \sum_{t=1}^4 V_{ik}^t ; \quad (12)$$

$$(k = 1, 2, 3)$$

the total supply of skim milk for drying:

$$D_3 = \sum_{i=1}^I \sum_{t=1}^4 X_{i1}^t ; \quad (13)$$

the total supply of skim milk by reconstitution:

$$D_4 = \sum_{i=1}^I \sum_{t=1}^4 X_{i3}^t ; \quad (14)$$

and finally the total supply of milk for cheese-making:

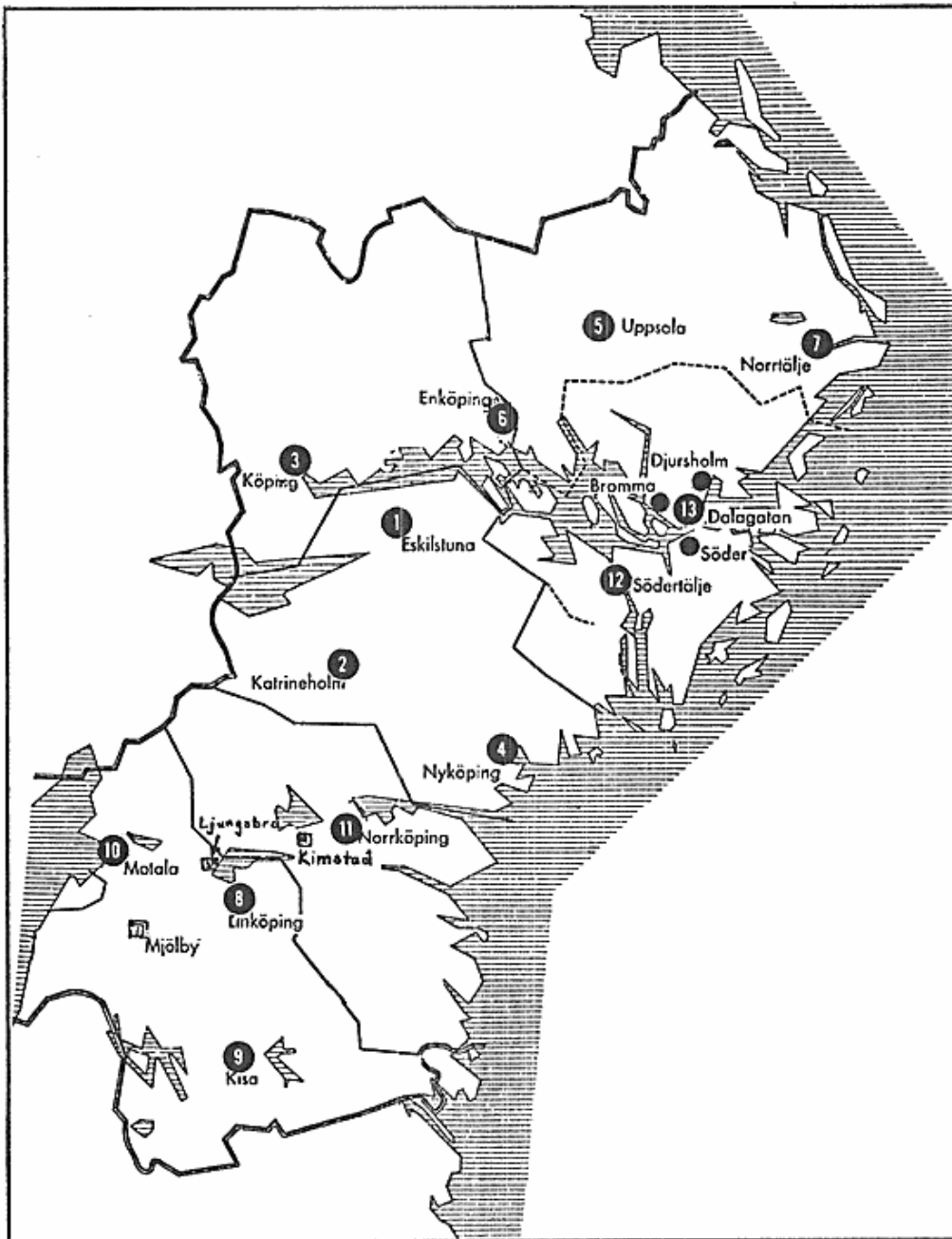
$$D_5 = \sum_{i=1}^I \sum_{t=1}^4 (X_{i4}^t + X_{i5}^t) ; \quad (15)$$

##### 5. Two Case Studies – July and October 1964

In 1964 the Mjölkcenralen was operating thirteen main dairies located in central Sweden. Two of these were supplied with facilities for milk drying, six with capacities for making cheese and two with butter production facilities. Twelve of the thirteen dairies nearly always had a milk surplus when the thirteenth (in Stockholm) had a considerably shortage of milk. All these sites are shown in Figure 2. Their production and transportation capacities are shown in Table 1. Four of dairies were situated relatively close to the three outside production sites in Kimstad (Semper), Ljungsbro (Cloetta), and Mjölby (a milk sugar plant). However, these four dairies had a highly developed production of cheese (Linköping, Kisa and Motala) or dried milk (Norrköping). Therefore, the trivial solution for meeting the demand at these three outside sites by skim milk from the closest dairies would perhaps result in some unused capacity at these dairies, either for milk drying or for making cheese. Therefore, one question is how to weight the transportation costs to the three production sites in relation to e. g. the costs of overtime in cheese production dairies.

Another question is which dairies can serve to eliminate the milk shortage in Stockholm. Moreover, a solution must determine which production or transportation quantities shall fluctuate in size over the week, since the total milk supply is expected to change.

In these two studies, one was made covering a week in the month (July) with maximal milk surplus. At that time the drying was expected to use



*Figure 2: The thirteen dairies and the three outside production places in Kimstad, Ljungsbro, and Mjölby.*

maximum capacities in order to take care of the great surplus. The other study was covering October, when dried milk had to be reconstituted since milk production was low.

Dairies	Production alternatives						Transportation alternatives			
	a. Skim milk for drying	b. Skim milk for feed	c. Reconstitution	d. Cheesemaking to normal costs	e. Cheesemaking to higher costs	f. Butter	a. Raw milk to other dairies	b. Raw milk from other dairies	c. Skim milk to the outside production places	d. Cream for the butter-making dairies
1. Eskilstuna		x	x				x	x	x	x
2. Katrineholm	x	x	x			x	x	x	x	x
3. Köping		x	x	x	x		x	x	x	x
4. Nyköping		x	x	x	x		x	x	x	x
5. Uppsala		x	x				x	x	x	x
6. Enköping		x	x	x	x		x	x	x	x
7. Norrtälje		x	x				x	x	x	x
8. Linköping		x	x	x	x		x	x	x	x
9. Kisa		x	x	x	x		x	x	x	x
10. Motala		x	x	x	x		x	x	x	x
11. Norrköping	x	x	x			x	x	x	x	x
12. Södertälje		x	x				x	x	x	x
13. Stockholm								x		

Table 1: Production and transportation alternatives for the 13 dairies.

The results obtained showed, that the high production costs for Sundays meant that the milk was put into storage as much as possible. The amount of milk in these storages was then reduced gradually up to the end of the week. But during the week in July the storages were filled to maximum capacity on Sundays so that these days had to be used for production too.

The four dairies (in Linköping, Kisa, Motala and Norrköping) close to the three outside production sites (in Ljungsbro, Kimstad and Mjölby)

did supply their total demand for skim milk in October. But in July their own production capacities were used maximally. Therefore the outside sites were supplied optimally with milk from dairies far away (as from Katrineholm, Uppsala and Norrtälje).

The alternative of transporting the milk surplus *between* the dairies was used to a very small degree. And when it was used, it was only between dairies close to one another (as from Södertälje, Nyköping or Enköping to Katrineholm).

The capacities for milk drying were used almost maximally. However, the solutions did show overcapacity in making cheese. During October, only two dairies (Kisa and Motala) had to be used for making cheese. In July, five of the six dairies were used (Köping, Nyköping, Enköping, Kisa and Motala, but not Linköping).

The linear programming model constructed for these case studies gave about 700 variables, 425 restrictions and 3200 non-zero elements. Each optimization took about 8 minutes on the Univac 1107 computer (performed in 1966).

## 6. Conclusions

In this study a model for the short term operation of a set of dairies has been constructed. The problem was stated as follows: given production and storage capacities, surplus of raw milk and demand for milk products at a set of sites in a region, given capacities for transportation between these sites, how will the operation be scheduled over a relatively short time period. The problem was formulated in a linear programming model and solved for a milk production firm (with thirteen plants) in Sweden under »peak« supply in July and normal supply in October.

The results show that milk shortages do not always have to be filled from the closest site that has a surplus. Instead, the alternative costs of transportation, production and storing have to be weighed.

Moreover, even production is not the most profitable way when there is fluctuating supply or demand. A short-term operation of a set of plants, will receive benefits if a total plan is scheduled and the plants are well coordinated as if they were pieces in an integrated system for the firm. We have shown that such a system can be operated efficiently by the use of linear programming.